

Table S1. Lime requirement estimated from the traditional methods that were added to all soils (n = 22) used in the incubation study.

Soil	Method ^{1/}						pHOM
	M ^{x+}	BSAT	MG ₁	MG ₂	SMP	HAI	
	g dm ⁻³						
LVAd ₁	0.27	1.89	1.20	0.87	2.55	3.18	5.80
LVd ₁	0.33	0.95	1.45	1.21	1.60	1.62	2.48
LAW	0.42	1.64	1.77	1.60	2.20	2.51	5.33
LVAd ₂	0.65	1.39	1.89	1.47	1.90	2.44	3.52
LVAa	1.45	3.02	3.45	2.80	5.80	4.89	11.93
LVAd ₃	0.40	0.76	1.45	1.21	0.90	1.34	1.20
LVd ₂	0.16	1.46	0.39	0.10	2.20	2.94	4.35
LVAd ₄	1.48	2.79	3.61	3.20	3.35	4.20	8.27
LVAd ₅	0.67	2.21	2.07	1.82	3.35	3.43	7.61
LVd ₃	1.23	2.31	2.87	2.40	3.35	3.72	10.27
LVd ₄	0.89	1.86	1.62	1.04	3.35	3.46	6.52
PVAd ₁	0.34	0.95	0.95	0.59	1.90	1.94	3.00
CXbd	0.85	1.35	2.32	2.08	1.35	2.06	3.84
LVd ₅	0.98	1.80	2.39	2.13	3.80	2.77	8.91
LVw	0.56	1.48	1.80	1.66	1.10	2.23	5.18
PVAd ₂	0.16	1.29	0.66	0.46	1.35	2.39	2.80
LVAd ₆	0.54	0.60	0.98	0.76	0.90	1.33	1.68
LAd ₁	0.80	1.17	1.59	1.45	1.60	1.85	3.79
LVd ₆	0.47	0.97	1.30	1.23	0.70	1.49	1.96
LAd ₂	0.75	1.54	1.39	1.27	2.20	2.44	4.08
RQo ₁	0.62	0.43	0.42	0.27	0.90	1.35	2.12
RQo ₂	0.35	0.41	0.98	0.99	0.00	0.59	0.30
Lowest	0.16	0.41	0.10	0.39	0.00	0.59	0.30
Highest	1.48	3.02	3.20	3.61	5.80	4.89	11.93
Mean	0.65	1.47	1.39	1.66	2.11	2.46	4.77
CV (%)	57.7	47.6	62.3	47.5	62.5	42.5	64.2

⁽¹⁾ See Table 2 of main text for reference of traditional methods to estimate lime requirement.

Table S2. Regression equations relating soil pH (\hat{y}) as a function of the lime rates (x) for all soils (n = 22) used in the incubation study.

Soil	Equation	R ²
LVAd ₁	$\hat{y} = 4.92 + 0.2641^{**} x - 0.0113^{**} x^2$	0.961
LVd ₁	$\hat{y} = 5.37 - 0.0339 x + 0.0358^{**} x^2$	0.929
Law	$\hat{y} = 4.92 + 0.3298^{**} x - 0.0143^{**} x^2$	0.968
LVAd ₂	$\hat{y} = 5.13 + 0.1870^{**} x - 0.0074^{\circ} x^2$	0.950
LVAa	$\hat{y} = 5.0 + 0.140^{**} x - 0.0029^{**} x^2$	0.962
LVAd ₃	$\hat{y} = 5.64 + 0.1543^{**} x + 0.0435^{**} x^2$	0.858
LVd ₂	$\hat{y} = 5.56 - 0.0133 x + 0.0147^{**} x^2$	0.820
LVAd ₄	$\hat{y} = 5.37 + 0.1034^{**} x - 0.0019^{**} x^2$	0.795
LVAd ₅	$\hat{y} = 5.15 + 0.2288^{**} x - 0.0064^{**} x^2$	0.963
LVd ₃	$\hat{y} = 4.69 + 0.2274^{**} x - 0.0066^{**} x^2$	0.985
LVd ₄	$\hat{y} = 5.54 + 0.1023^{**} x$	0.948
PVAd ₁	$\hat{y} = 5.52 + 0.3841^{**} x - 0.0139^{*} x^2$	0.972
CXbd	$\hat{y} = 4.95 + 0.3122^{**} x - 0.0142^{**} x^2$	0.850
LVd ₅	$\hat{y} = 5.35 + 0.1973^{**} x - 0.0056^{**} x^2$	0.981
LVw	$\hat{y} = 4.83 + 0.3811^{**} x - 0.0165^{**} x^2$	0.953
PVAd ₂	$\hat{y} = 5.33 + 0.1961^{**} x$	0.938
LVAd ₆	$\hat{y} = 4.90 + 0.2659^{**} x$	0.984
LAd ₁	$\hat{y} = 5.15 + 0.3847^{**} x - 0.0198^{**} x^2$	0.805
LVd ₆	$\hat{y} = 5.42 + 0.3526^{**} x$	0.921
LAd ₂	$\hat{y} = 5.45 + 0.2350^{**} x - 0.0095^{**} x^2$	0.936
RQo ₁	$\hat{y} = 5.31 + 0.3405^{**} x - 0.0216^{*} x^2$	0.971
RQo ₂	$\hat{y} = 5.10 + 1.3974^{**} x - 0.2901^{**} x^2$	0.818

[°]: 0.10 > p ≥ 0.05. *: 0.05 > p ≥ 0.01. **: p < 0.01.

Table S3. Regression equations relating exchangeable acidity (\hat{y}) as a function of the lime rates (x) for all soils ($n = 22$) used in the incubation study.

Soil	Equation	Experimental space V	R ²
LVAd ₁	$\hat{y} = 0.25 e^{-0.4147^* x^2}$	$0 \leq x \leq 3.8$	0.984
	$\hat{y} = \bar{y} = 0$	$x > 3.8$	
LVd ₁	$\hat{y} = 0.25 e^{-0.8738^{**} x}$	$0 \leq x \leq 2.9$	0.996
	$\hat{y} = \bar{y} = 0.01$	$x > 2.9$	
LAW	$\hat{y} = 0.50 e^{-1.0517^{**} x}$	$0 \leq x \leq 3.2$	0.999
	$\hat{y} = \bar{y} = 0$	$x > 3.2$	
LVAd ₂	$\hat{y} = 0.72 e^{-0.6548^{**} x}$	$0 \leq x \leq 4.9$	0.996
	$\hat{y} = \bar{y} = 0$	$x > 4.9$	
LVAA	$\hat{y} = 1.30 e^{-0.4880^{**} x}$	$0 \leq x \leq 9.8$	0.999
	$\hat{y} = \bar{y} = 0.01$	$x > 9.8$	
LVAd ₃	$\hat{y} = 0.42 e^{-1.4268^{**} x}$	$0 \leq x \leq 2.4$	0.984
	$\hat{y} = \bar{y} = 0$	$x > 2.4$	
LVd ₂	$\hat{y} = 0.08 e^{-0.4098^{**} x}$	$0 \leq x \leq 4.4$	0.981
	$\hat{y} = \bar{y} = 0.01$	$x > 4.4$	
LVAd ₄	$\hat{y} = 1.78 e^{-0.5114^{**} x}$	$0 \leq x \leq 7.2$	0.997
	$\hat{y} = \bar{y} = 0$	$x > 7.2$	
LVAd ₅	$\hat{y} = 0.61 e^{-0.9581^{**} x}$	$0 \leq x \leq 3.6$	0.999
	$\hat{y} = \bar{y} = 0.01$	$x > 3.6$	
LVd ₃	$\hat{y} = 1.53 e^{-0.4460^{**} x}$	$0 \leq x \leq 6.7$	0.993
	$\hat{y} = \bar{y} = 0.02$	$x > 6.7$	
LVd ₄	$\hat{y} = 0.71 e^{-0.6445^{**} x}$	$0 \leq x \leq 6.7$	0.996
	$\hat{y} = \bar{y} = 0$	$x > 6.7$	
PVAd ₁	$\hat{y} = 0.14 e^{-1.5177^{**} x}$	$0 \leq x \leq 1.9$	0.996
	$\hat{y} = \bar{y} = 0$	$x > 1.9$	
CXbd	$\hat{y} = 0.99 e^{-1.0975^{**} x}$	$0 \leq x \leq 4.1$	0.998
	$\hat{y} = \bar{y} = 0$	$x > 4.1$	
LVd ₅	$\hat{y} = 0.90 e^{-0.6476^{**} x}$	$0 \leq x \leq 4.8$	0.998
	$\hat{y} = \bar{y} = 0.01$	$x > 4.8$	
LVw	$\ln \hat{y} = -0.37 - 1.0925^{**} x$	$0 \leq x \leq 3.6$	0.999
	$\hat{y} = \bar{y} = 0$	$x > 3.6$	
PVAd ₂	$\ln \hat{y} = -2.68 - 1.4749^{**} x$	$0 \leq x \leq 1.3$	0.997
	$\hat{y} = \bar{y} = 0$	$x > 1.3$	
LVAd ₆	$\ln \hat{y} = -0.55 - 0.9344^{**} x$	$0 \leq x \leq 2.7$	0.995
	$\hat{y} = \bar{y} = 0$	$x > 2.7$	

Continue

Table S3. Continuation.

Soil	Equation	Experimental space V	R ²
LAd ₁	$\hat{y} = 0.84 e^{-1.1937^{**} x}$	$0 \leq x \leq 2.9$	0.989
	$\hat{y} = \bar{y} = 0$	$x > 2.9$	
LVd ₆	$\hat{y} = 0.32 e^{-1.5522^{**} x}$	$0 \leq x \leq 1.9$	0.993
	$\hat{y} = \bar{y} = 0$	$x > 1.9$	
LAd ₂	$\hat{y} = 0.01 + 0.4926^{**} e^{-x}$	$0 \leq x \leq 4.4$	0.991
	$\hat{y} = \bar{y} = 0.01$	$x > 4.4$	
RQo ₁	$\hat{y} = 0.51 e^{-0.9467^{**} x}$	$0 \leq x \leq 2.7$	0.988
	$\hat{y} = \bar{y} = 0$	$x > 2.7$	
RQo ₂	$\hat{y} = 0.29 e^{-2.5541^{**} x}$	$0 \leq x \leq 1.2$	0.994
	$\hat{y} = \bar{y} = 0$	$x > 1.2$	

*: $0.05 > p \geq 0.01$. **: $p < 0.01$.

Table S4. Regression equations relating potential acidity (\hat{y}) as a function of the lime rates (x) for all soils ($n = 22$) used in the incubation study.

Soil	Equation	R ²
LVA _{d1}	$\hat{y} = 7.40 e^{-0.11^{**} x}$	0.984
LV _{d1}	$\hat{y} = 5.48 - 0.5419^{**} x$	0.986
LA _w	$\hat{y} = 6.43 e^{-0.1382^{**} x}$	0.984
LVA _{d2}	$\hat{y} = 7.46 e^{-0.1107^{**} x}$	0.997
LVA _a	$\hat{y} = 12.06 e^{-0.0788^{**} x}$	0.997
LVA _{d3}	$\hat{y} = 3.32 e^{-0.2688^{**} x}$	0.990
LV _{d2}	$\hat{y} = 6.20 e^{-0.1106^{**} x}$	0.981
LVA _{d4}	$\hat{y} = 11.51 e^{-0.0879^{**} x}$	0.993
LVA _{d5}	$\hat{y} = 9.08 e^{-0.1301^{**} x}$	0.997
LV _{d3}	$\hat{y} = 11.39 e^{-0.0803^{**} x}$	0.996
LV _{d4}	$\hat{y} = 8.48 e^{-0.1042^{**} x}$	0.991
PVA _{d1}	$\hat{y} = 4.28 e^{-0.2639^{**} x}$	0.995
CX _{bd}	$\hat{y} = 5.04 e^{-0.2187^{**} x}$	0.997
LV _{d5}	$\hat{y} = 10.25 e^{-0.1067^{**} x}$	0.997
LV _w	$\hat{y} = 5.88 e^{-0.1863^{**} x}$	0.993
PVA _{d2}	$\hat{y} = 5.43 e^{-0.17^{**} x}$	0.986
LVA _{d6}	$\hat{y} = 4.83 - 0.6665^{**} x$	0.996
LA _{d1}	$\hat{y} = 4.61 e^{-0.2530^{**} x}$	0.997
LV _{d6}	$\hat{y} = 3.39 e^{-0.2878^{**} x}$	0.978
LA _{d2}	$\hat{y} = 5.72 e^{-0.1652^{**} x}$	0.988
RQ ₀₁	$\hat{y} = 4.39 e^{-0.2319^{**} x}$	0.981
RQ ₀₂	$\hat{y} = 0.38 + 1.2324^{**} e^{-x}$	0.965

** : $p < 0.01$.

Table S5. Regression equations relating exchangeable calcium (\hat{y}) as a function of the lime rates (x) for all soils ($n = 22$) used in the incubation study.

Soil	Equation	R ²
LVAd ₁	$\hat{y} = 0.24 + 0.5833^{**} x - 0.0165^{**} x^2$	0.996
LVD ₁	$\hat{y} = 0.01 + 0.6702^{**} x - 0.0427^{**} x^2$	0.991
LAW	$\hat{y} = -0.08 + 0.5908^{**} x - 0.0199^{**} x^2$	0.998
LVAd ₂	$\hat{y} = 0.38 + 0.4740^{**} x$	0.991
LVAa	$\hat{y} = 0.32 + 0.5845^{**} x - 0.0109^{**} x^2$	0.990
LVAd ₃	$\hat{y} = 0.16 + 0.5872^{**} x$	0.996
LVD ₂	$\hat{y} = 1.75 + 0.5268^{**} x - 0.0189^{**} x^2$	0.997
LVAd ₄	$\hat{y} = -0.12 + 0.7129^{**} x - 0.0191^{**} x^2$	0.996
LVAd ₅	$\hat{y} = 0.0 + 0.6417^{**} x - 0.0226^{**} x^2$	0.959
LVD ₃	$\hat{y} = 0.41 + 0.4981^{**} x - 0.0097^{**} x^2$	0.994
LVD ₄	$\hat{y} = 1.10 + 0.5303^{**} x - 0.0153^{**} x^2$	0.996
PVAd ₁	$\hat{y} = 0.49 + 0.4300^{**} x$	0.980
CXbd	$\hat{y} = -0.05 + 0.6694^{**} x - 0.0256^{**} x^2$	0.989
LVD ₅	$\hat{y} = 0.09 + 0.5231^{**} x - 0.0130^{**} x^2$	0.994
LVw	$\hat{y} = -0.05 + 0.6026^{**} x - 0.0224^{**} x^2$	0.997
PVAd ₂	$\hat{y} = 1.43 + 0.4318^{**} x$	0.989
LVAd ₆	$\hat{y} = 0.67 + 0.4282^{**} x$	0.973
LAd ₁	$\hat{y} = -0.06 + 0.6680^{**} x - 0.0325^{**} x^2$	0.998
LVD ₆	$\hat{y} = 0.09 + 0.3860^{**} x$	0.978
LAd ₂	$\hat{y} = 0.04 + 0.5398^{**} x - 0.0207^{*} x^2$	0.993
RQ ₀₁	$\hat{y} = 0.55 + 0.4078^{**} x$	0.983
RQ ₀₂	$\hat{y} = -0.04 + 0.3839^{**} x$	0.977

*: $0.05 > p \geq 0.01$. **: $p < 0.01$.

Table S6. Regression equations relating exchangeable magnesium (\hat{y}) as a function of the lime rates (x) for all soils (n = 22) used in the incubation study.

Soil	Equation	R ²
LVA _{d1}	$\hat{y} = 0.24 + 0.1979^{**} x - 0.0094^{**} x^2$	0.988
LVD ₁	$\hat{y} = 0.09 + 0.1835^{**} x - 0.0106^{**} x^2$	0.988
LAW	$\hat{y} = 0.0 + 0.1881^{**} x - 0.0109^{**} x^2$	0.995
LVA _{d2}	$\hat{y} = 0.07 + 0.1743^{**} x - 0.0059^{**} x^2$	0.995
LVA _a	$\hat{y} = 0.10 + 0.2155^{**} x - 0.0071^{**} x^2$	0.993
LVA _{d3}	$\hat{y} = 0.01 + 0.2107^{**} x - 0.0224^{**} x^2$	0.999
LVD ₂	$\hat{y} = 0.83 + 0.1311^{**} x - 0.0045^{**} x^2$	0.951
LVA _{d4}	$\hat{y} = -0.03 + 0.2157^{**} x - 0.0075^{**} x^2$	0.997
LVA _{d5}	$\hat{y} = 0.08 + 0.2240^{**} x - 0.0115^{**} x^2$	0.934
LVD ₃	$\hat{y} = 0.32 + 0.1831^{**} x - 0.0057^{**} x^2$	0.996
LVD ₄	$\hat{y} = 0.08 + 0.1848^{**} x - 0.0083^{**} x^2$	0.998
PVA _{d1}	$\hat{y} = 0.47 + 0.1499^{**} x - 0.0134^{**} x^2$	0.995
CXbd	$\hat{y} = -0.02 + 0.2262^{**} x - 0.0191^{**} x^2$	0.992
LVD ₅	$\hat{y} = 0.03 + 0.2079^{**} x - 0.0080^{**} x^2$	0.988
LVw	$\hat{y} = -0.02 + 0.2034^{**} x - 0.0144^{**} x^2$	0.996
PVA _{d2}	$\hat{y} = 1.28 + 0.1800^{**} x - 0.0187^{**} x^2$	0.804
LVA _{d6}	$\hat{y} = 0.36 + 0.1040^{**} x$	0.978
LAd ₁	$\hat{y} = 0.04 + 0.2131^{**} x - 0.0207^{**} x^2$	0.994
LVD ₆	$\hat{y} = 0.02 + 0.2210^{**} x - 0.0249^{**} x^2$	0.991
LAd ₂	$\hat{y} = 0.07 + 0.1691^{**} x - 0.0094^{**} x^2$	0.986
RQ ₀₁	$\hat{y} = 0.70 + 0.1692^{**} x - 0.0140^{**} x^2$	0.943
RQ ₀₂	$\hat{y} = 0.0 + 0.1432^{**} x - 0.0363^{**} x^2$	0.853

** : p < 0.01.