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Supplementary Material to "On the development of the Navier–Stokes equation by Navier"

On the conditions at the interface of a fluid in contact with a solid wall

Although it now appears to have been definitely accepted that the fluid layer immediately in contact with a solid wall has no relative velocity to the wall, in the 19th century this issue was not settled and motivated intense experimental research by many prominent investigators[1].

In the 19th century, there were essentially three different hypotheses as to the conditions at the surface of a solid wall in contact with the fluid in motion. According to the first, attributed to Coulomb, which is based on evidences experimentally obtained by him, the velocity is the same at a solid wall as that of the solid wall itself, and changes continuously in the fluid, which has everywhere the same properties. This is what we call a 'strict non-slip boundary condition'. The second hypothesis, put forward very clearly by Girard from experiments on the flow of liquids through capillaries, says a very thin layer of fluid remains completely attached to the walls, and that the rest of the fluid slips over it. He also supposed that if the walls are of the same material, the layer has a constant thickness, so that its surface presents to the flow the same irregularities as those of the wall itself, and that the thickness of the layer depends on the temperature. The layer thickness would be different for different liquids or different wall materials. It would become zero for liquids which do no wet the walls, as for mercury in glass tubes; in such cases, he supposed that the liquid slipped over the surface. As we have seen, the hypothesis of a strict non-slip boundary condition was embraced by Navier in the 1st memoir but later put in doubt, which motivated him to write the 2nd memoir. The third hypothesis has been attributed to Navier, but apparently was never presented in such terms by him. It was proposed from the expression

[1] The following contains summarized material from "Note on the conditions at the surface of contact of a fluid with a solid body", that appeared at the end of the 2nd volume of Modern Development in Fluid Mechanics (edited by S. Goldstein), Oxford, 1938.

developed by Navier in the 2nd memoir for the boundary conditions at the walls, which assumes that there is a slipping at the wall, and this slipping is resisted by a force proportional to the relative velocity. The balance of forces at the interface leads to $Eu = -\epsilon \frac{du}{dz}$, which indicates that the stress applied by the fluid at the solid wall, given by $\epsilon \frac{du}{dz}$, is balanced by the stress applied by the wall to the fluid in contact with it, given by Eu, such that ε/E is a length. This length would be zero if there was no slip. As we have seen earlier, Navier explained Girard's result by the application of this boundary condition.

From a molecular point of view, Maxwell seems to be one investigator who elaborated further on Navier's boundary condition. According to Maxwell, based on the hypothesis that the stratum of gas nearest to a solid wall is in a very different state from the rest of the gas, and after some calculations, the slip takes place according to Navier's boundary condition, and the length ε/E is a moderate multiple of the mean free path. Thus at atmospheric pressure the slip would be negligible; for rarefied gases, however, it would be considerable. Further investigation on this issue by others led to the conclusion that if an aggregate of the molecules near a solid wall continues to have the properties of a gas, the fluid velocity must vanish at the wall, and that this velocity is practically the same as that of the wall at some short distance away. However, if a layer of fluid remains completely attached to a wall (Girard's assumption), or if slip takes place according to Navier's boundary condition, then another length scale, $l = \varepsilon/E$, must be considered in addition to the length scale d of the system (such as the tube diameter), and when the dimensions of the system are changed, force-coefficients and other non-dimensional quantities would depend on l/d. The conclusion is that unless in some curious way l varies in proportion to d, experiments have indicated that l is zero, or at most so small that its effects are negligible.

Stokes, in turn, was initially inclined to the first hypothesis (strict non-slip boundary condition), but hesitated between this hypothesis and Navier's after calculations on flow through tubes not in agreement with experiments known to him. However, reasoning that the existence of slip would imply that the friction between solid and fluid was of a different nature, and infinitely less than the friction between two layers of fluid, and also that the agreement with observation of results obtained on the assumption of non-slip was highly satisfactory, he later decided on the first hypothesis.

Poiseuille, in his memoir on the motion of blood, found a layer of stagnant blood at the walls of the containing vessel, and by observing the flow through glass tubes, with opaque bodies in suspension, found stagnant layers at the walls of thickness much less than any obtained by Girard. Hagen simply stated in his first paper that the velocity increases at a uniform rate from zero at the walls to a maximum in the middle; later he adopted the idea of a stagnant layer near the walls, but without slip, and found that in his tubes the layer had to be thinner than the "thinnest writing-paper".

Other 19th century investigators contributed to the subject, such as Darcy, who substantially agreed with Girard's hypothesis, and Helmholtz, who adopted Navier's hypothesis in his discussion of experiments conducted by others, concluding that there might be no slip for water in contact with glass, and that there was considerable slip for water in contact with a gilt surface. Couette, after discussing at length various experiments on the determination of viscosity, came to the conclusion that the relative velocity is actually zero at the boundary, but changes very rapidly in its neighbourhood.

Based on these accounts, it is possible to conclude that if slip takes place, or if there is a stagnant fluid layer, they would be too small or too thin to be observed or to make any observable difference in the results obtained by theoretical calculations. Gradually, however, the hypothesis of a strict non-slip boundary condition gained ground, as well as the idea that all parts of the fluid have the same properties, both of which are now generally accepted.