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Supplementary Material to "The Magnetic field for an n-cusped Epi- and Hypo-Cycloids loop current"

Appendix

A. Hypocycloid curve

A hypocycloid curve is a special plane curve generated by the trace of a fixed point P on a small circle that rolls within a larger circle. From the figure A1, the coordinates of the point P are:

$$x = (a-b)\,\cos\,\theta + b\,\cos\,\phi \tag{A.1}$$

$$y = (a-b)\,\sin\,\theta - b\,\sin\phi \tag{A.2}$$

From figure A1, the arc length $A'CP$ and $A'OA$ are equal

$$a\,\theta = b\,(\theta+\phi) \rightarrow \phi = \left(\frac{a-b}{b}\right)\theta \tag{A.3}$$

Replacing:

$$x = (a-b)\,\cos\,\theta + b\,\cos\left(\frac{a-b}{b}\,\theta\right) \tag{A.4}$$

$$y = (a-b)\,\sin\,\theta - b\,\sin\left(\frac{a-b}{b}\,\theta\right) \tag{A.5}$$

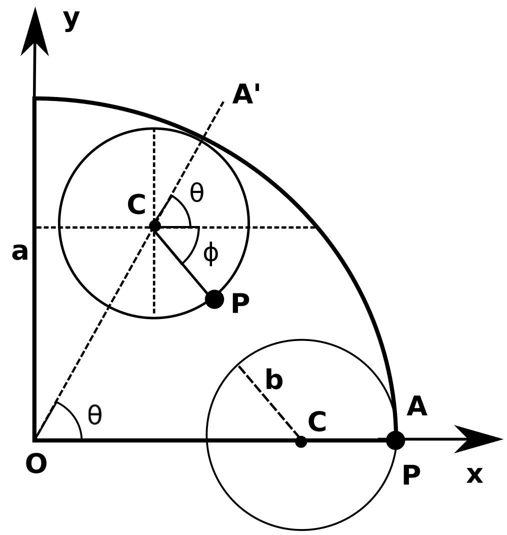


Figure A1: A small circle of radius b rolls inside a larger circle of radius a in a clockwise direction. The trace of a fixed point P in the smaller circle describe a hypocycloid curve.

B. Epicycloid curve

An epicycloid curve is a special plane curve generated by the trace of a fixed point P on a small circle that rolls outside a larger circle. From the figure B1, the coordinates of the point P are:

$$x = (a+b)\ \cos\ \theta - b\ \cos\ \phi \tag{B.1}$$

$$y = (a+b)\ \sin\ \theta - b\ \sin\phi \tag{B.2}$$

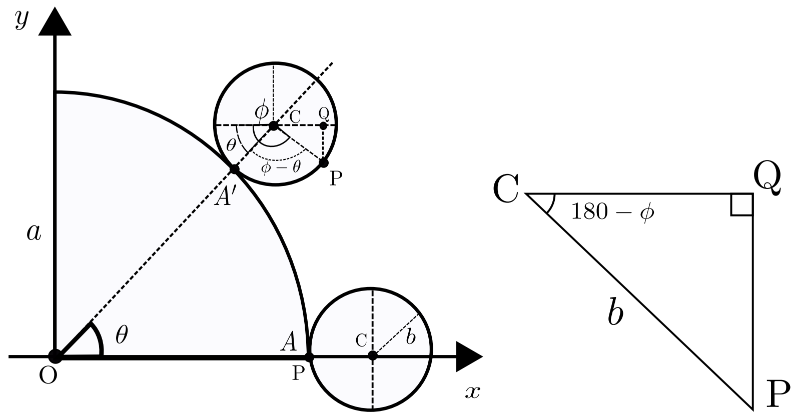
From figure B1, the arc length $A'CP$ and $A'OA$ are equal

$$a\,\theta = b\,(\phi - \theta) \rightarrow \phi = \left(\frac{a+b}{b}\right)\theta \tag{B.3}$$

Replacing:

$$x = (a+b)\ \cos\ \theta - b\ \cos\left(\frac{a+b}{b}\,\theta\right) \tag{B.4}$$

$$y = (a+b)\ \sin\ \theta - b\ \sin\left(\frac{a+b}{b}\,\theta\right) \tag{B.5}$$



(a) (b)

Figure B1: A small circle of radius b rolls outside a larger circle of radius a in a counterclockwise direction. The trace of a fixed point P in the smaller circle describe an epicycloid curve.