Supplementary Information

Coupled Kinetic and Rheological Analysis of Lactic Acid Production from Whey by *Kluyveromyces marxianus*

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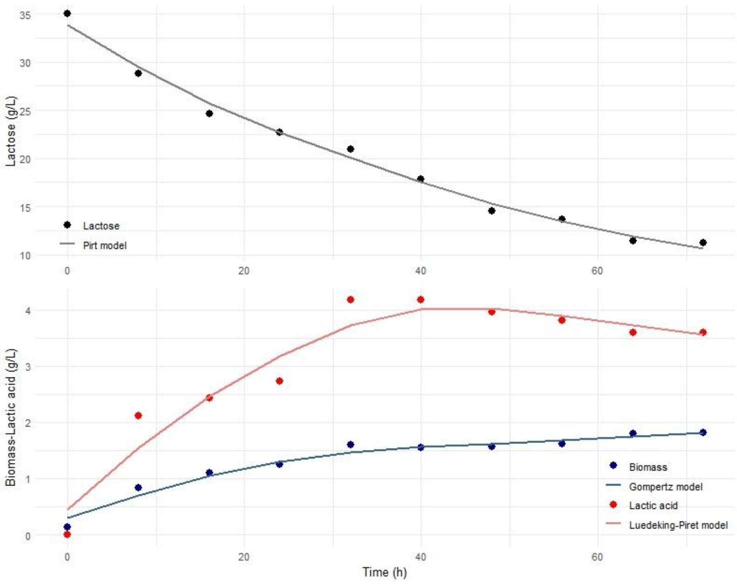

Figure S1. Kinetic adjustment using growth, production, and consumption models in lactic acid production for treatment 1.

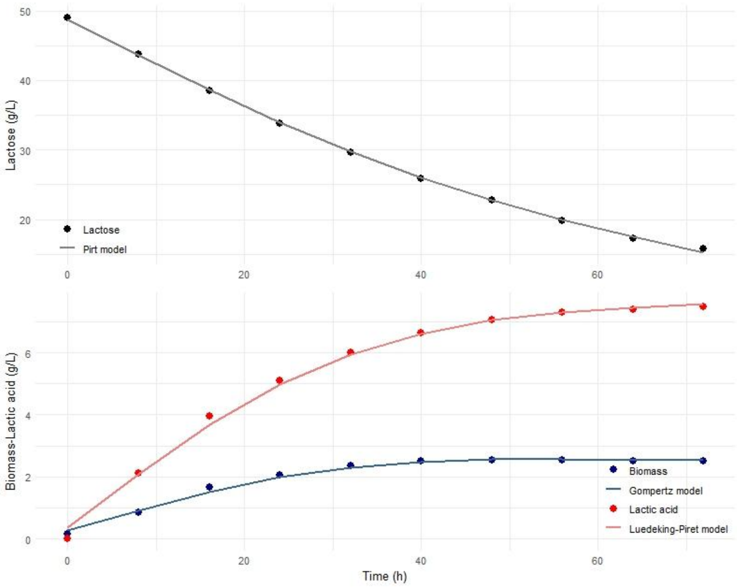

Figure S2. Kinetic adjustment using growth, production, and consumption models in lactic acid production for treatment 2.

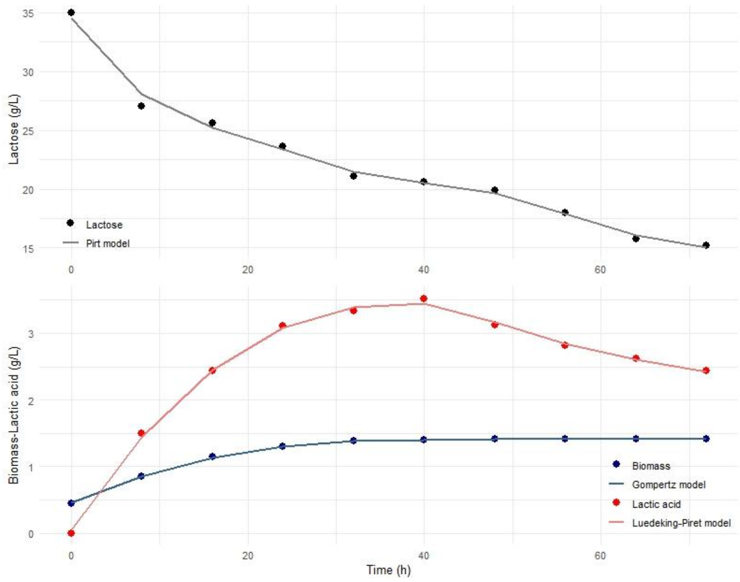

Figure S3. Kinetic adjustment using growth, production, and consumption models in lactic acid production for treatment 3.

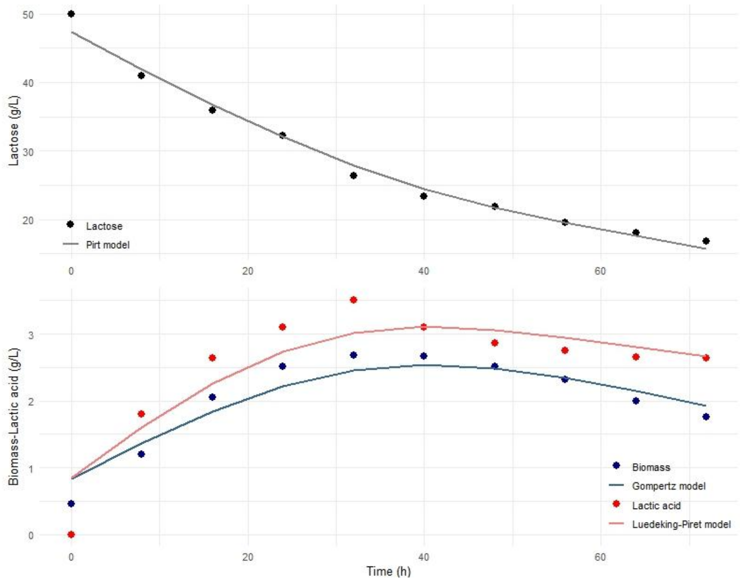

Figure S4. Kinetic adjustment using growth, production, and consumption models in lactic acid production for treatment 4.

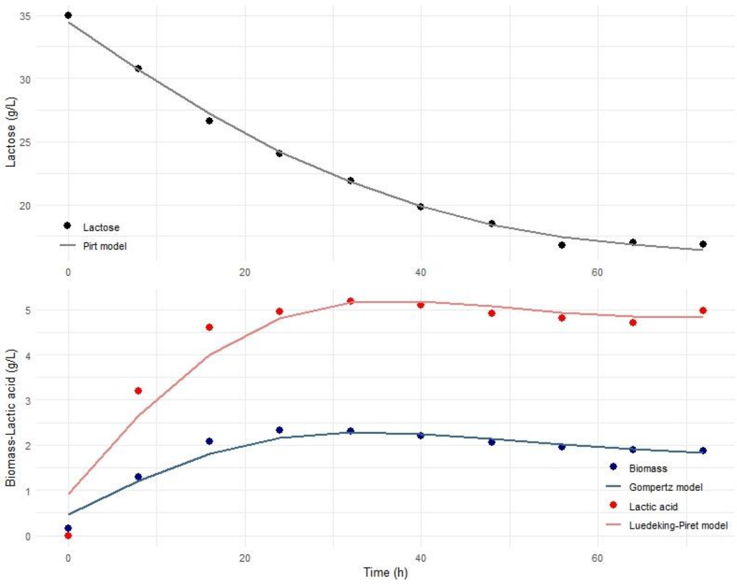

Figure S5. Kinetic adjustment using growth, production, and consumption models in lactic acid production for treatment 5.

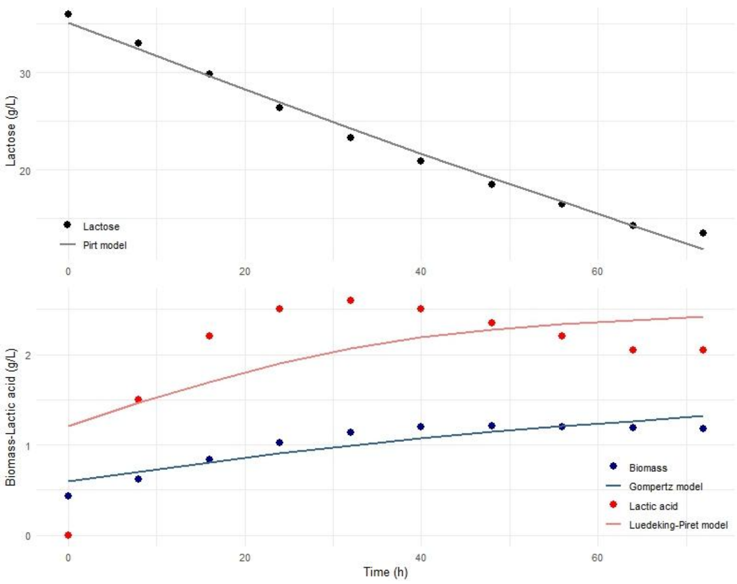

Figure S6. Kinetic adjustment using growth, production, and consumption models in lactic acid production for treatment 7.

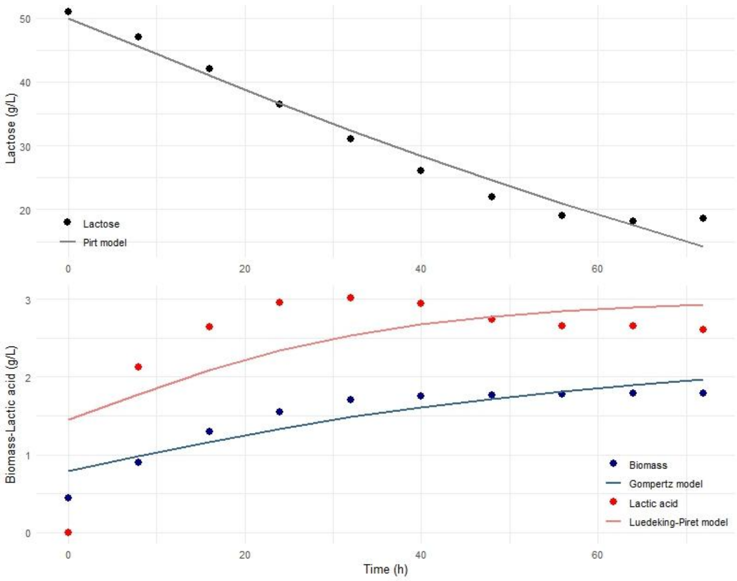

Figure S7. Kinetic adjustment using growth, production, and consumption models in lactic acid production for treatment 8.

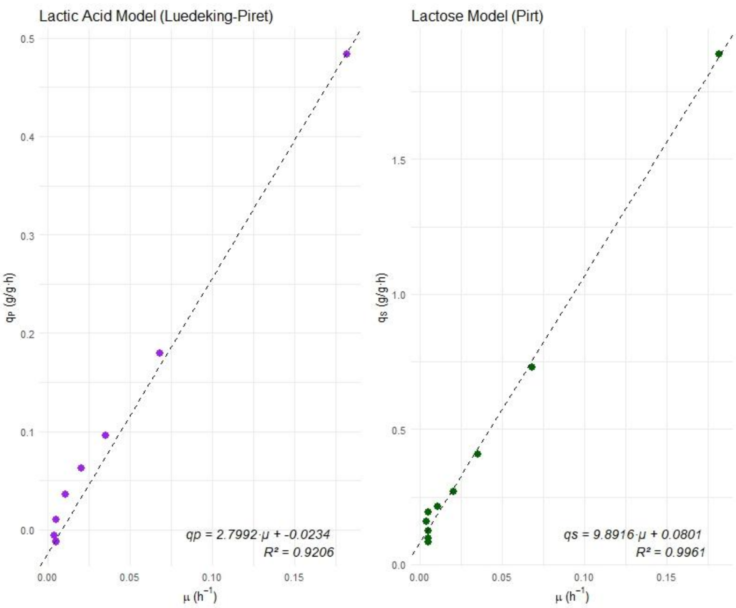

Figure S8. Validation of the Luedeking-Piret and Pirt growth models associated with lactic acid production and lactose consumption for treatment 1.

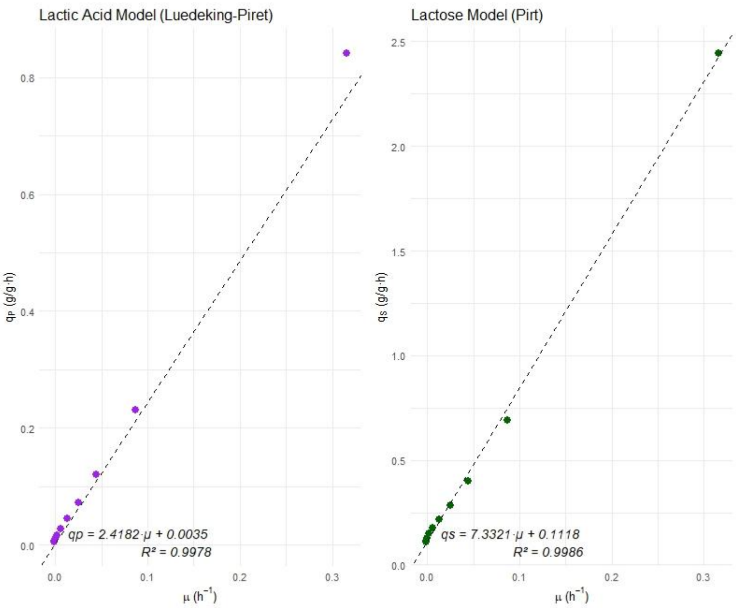

Figure S9. Validation of the Luedeking-Piret and Pirt growth models associated with lactic acid production and lactose consumption for treatment 2.

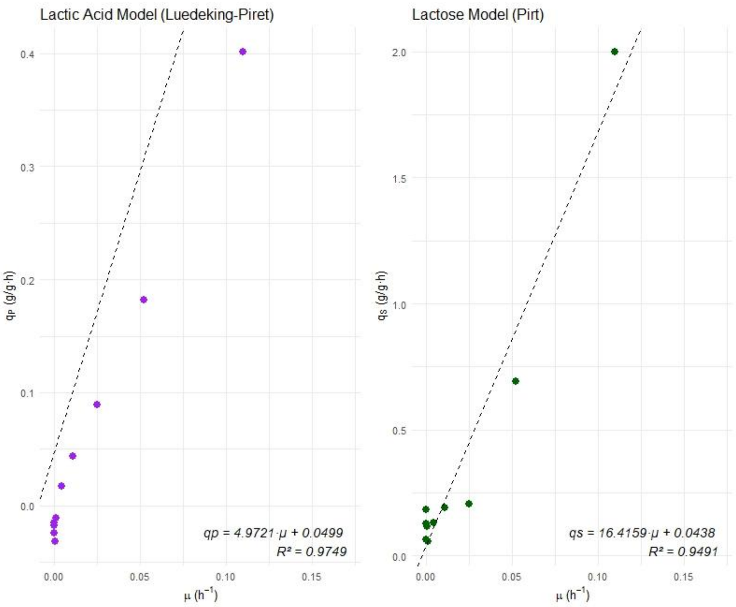

Figure S10. Validation of the Luedeking-Piret and Pirt growth models associated with lactic acid production and lactose consumption for treatment 3.

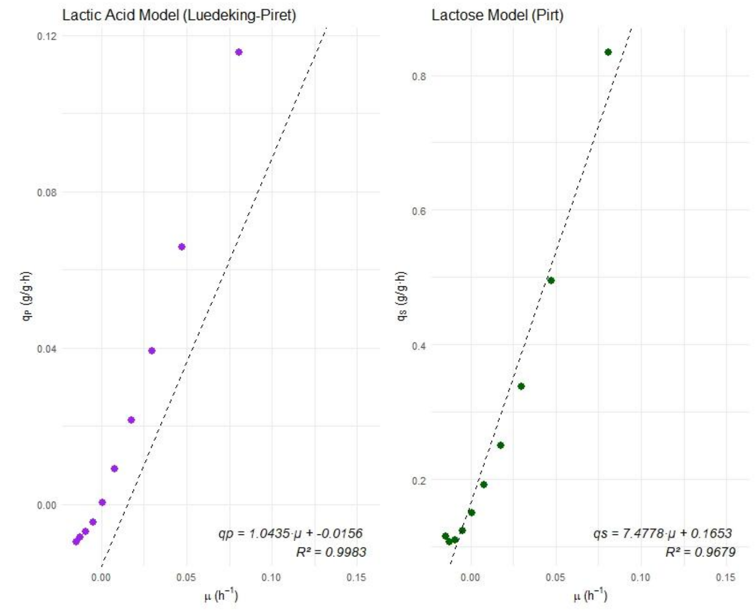

Figure S11. Validation of the Luedeking-Piret and Pirt growth models associated with lactic acid production and lactose consumption for treatment 4.

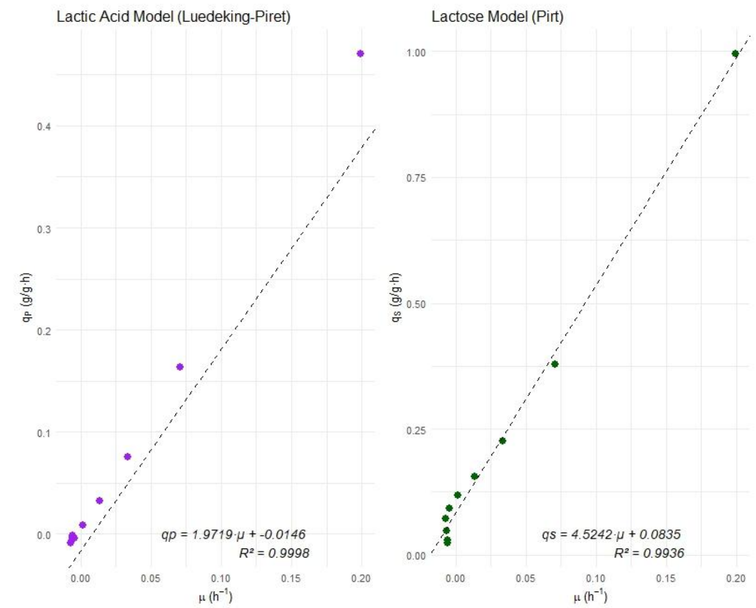

Figure S12. Validation of the Luedeking-Piret and Pirt growth models associated with lactic acid production and lactose consumption for treatment 5.

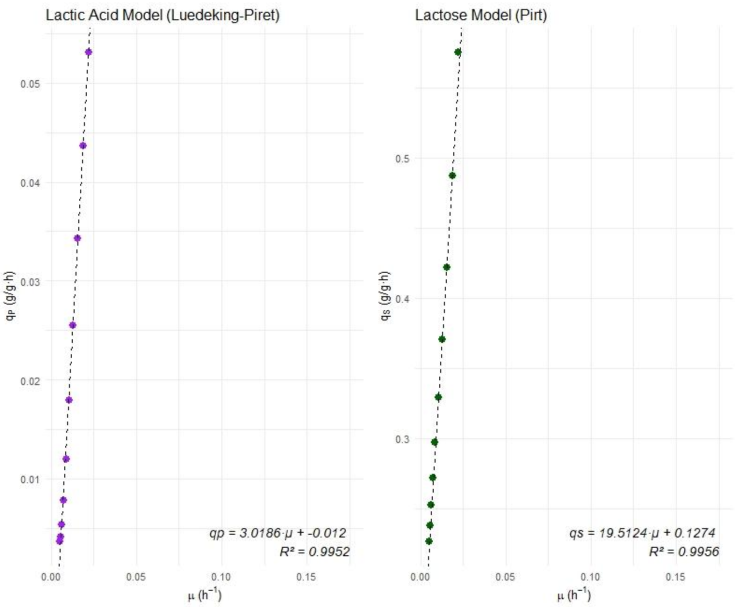

Figure S13. Validation of the Luedeking-Piret and Pirt growth models associated with lactic acid production and lactose consumption for treatment 7.

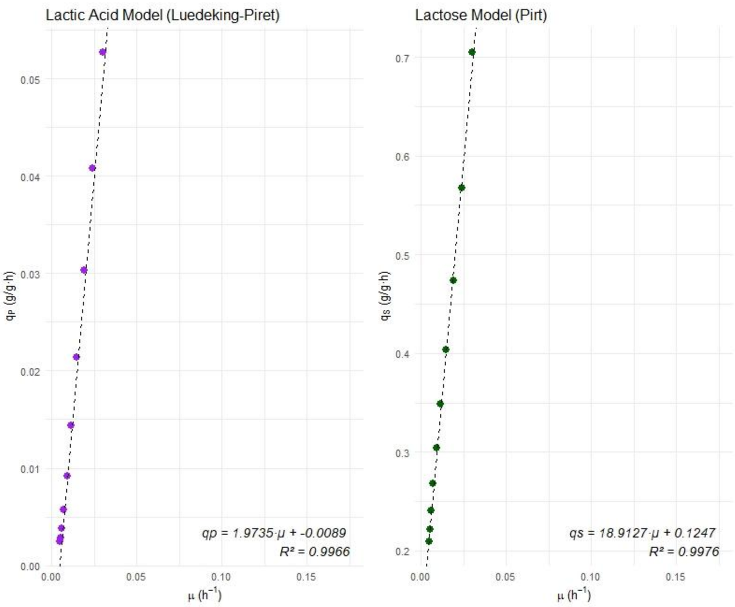

Figure S14. Validation of the Luedeking-Piret and Pirt growth models associated with lactic acid production and lactose consumption for treatment 8.

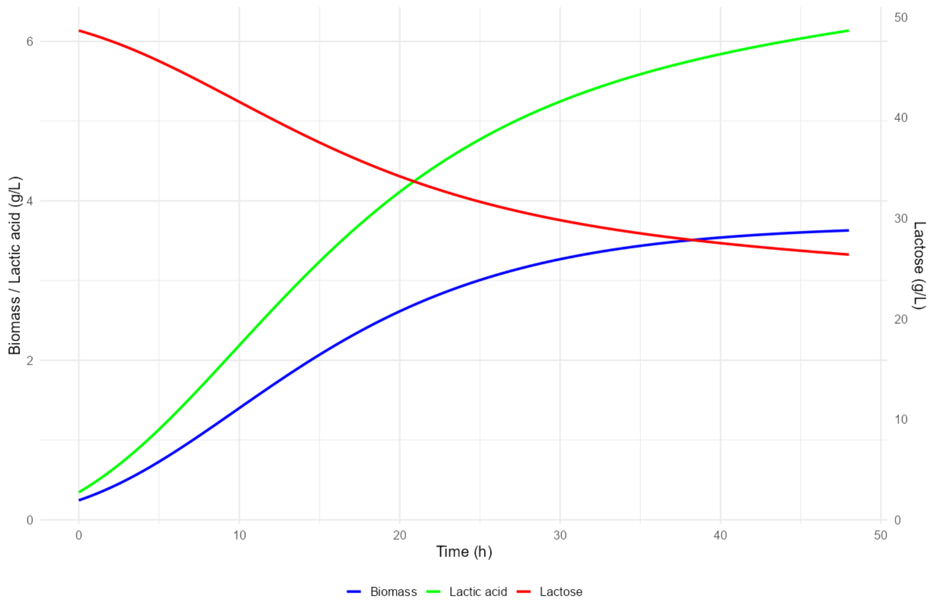

Figure S15. Simulation in Shiny of the effect of kinetic parameters on the growth of *K. marxianus*, lactose consumption, and lactic acid production.

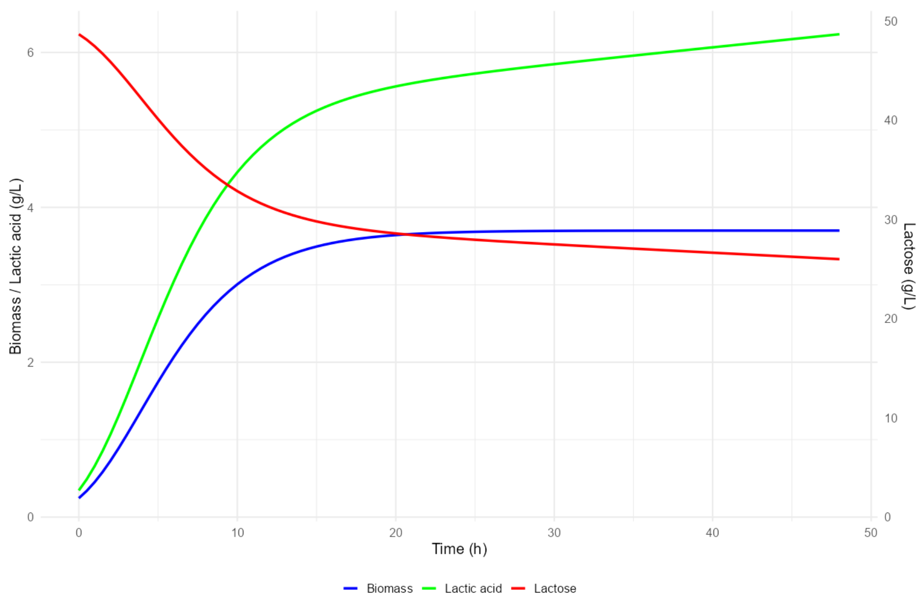

Figure S16. Simulation in Shiny of the effect of high specific growth rate on kinetic parameters on the growth of *K. marxianus*, lactose consumption, and lactic acid production.

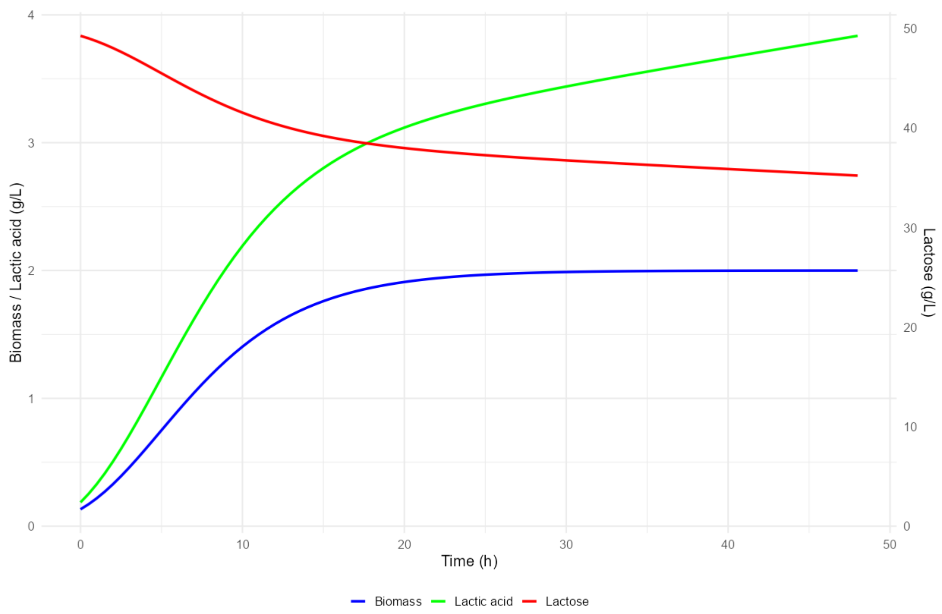

Figure S17. Simulation in Shiny of the effect of low biomass produced on kinetic parameters on the growth of *K. marxianus*, lactose consumption, and lactic acid production.

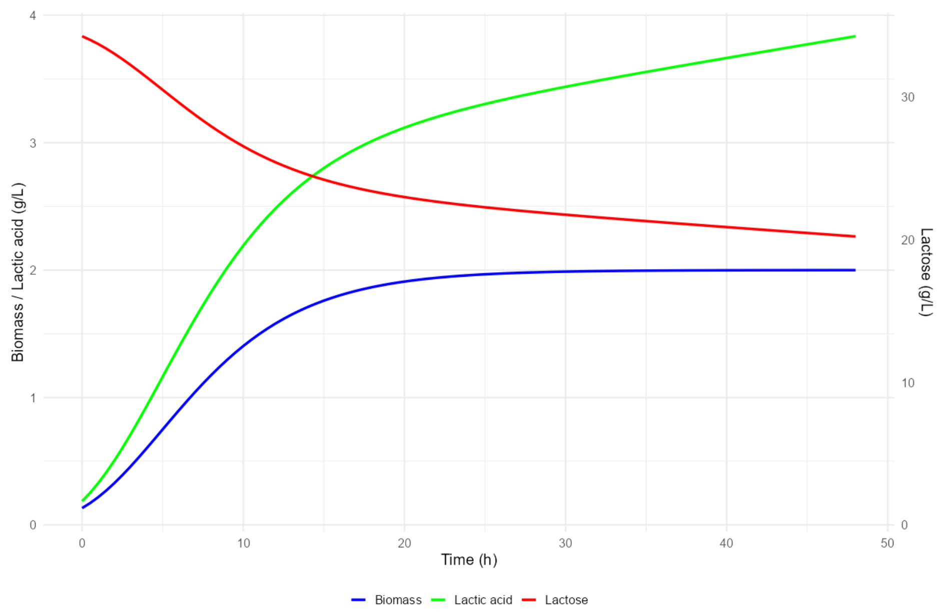

Figure S18. Simulation in Shiny of the effect of low biomass produced and low substrate concentration on kinetic parameters on the growth of *K. marxianus*, lactose consumption, and lactic acid production.

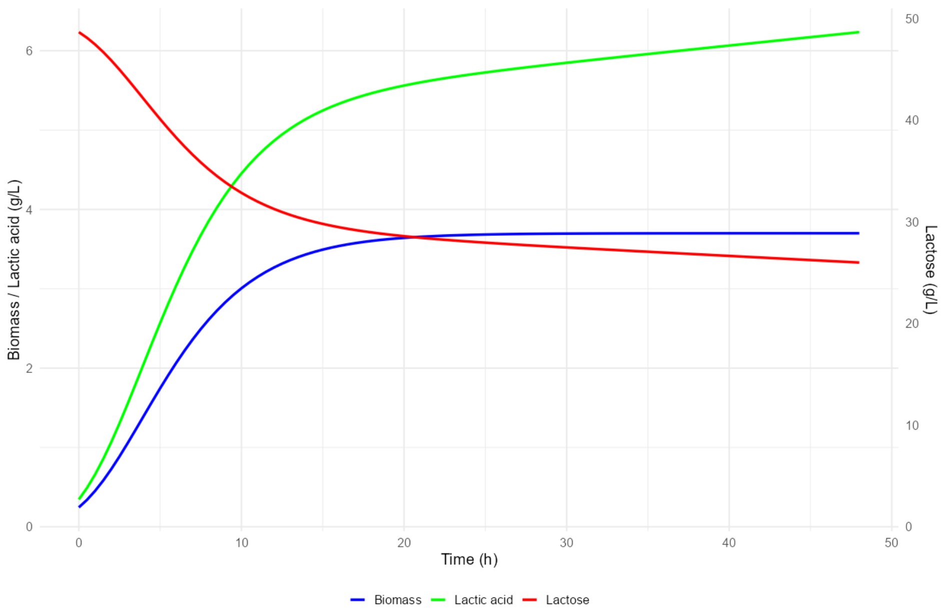

Figure S19. Simulation in Shiny of the effect of high biomass produced, high growth rate, and high substrate concentration on kinetic parameters on the growth of *K. marxianus*, lactose consumption, and lactic acid production.

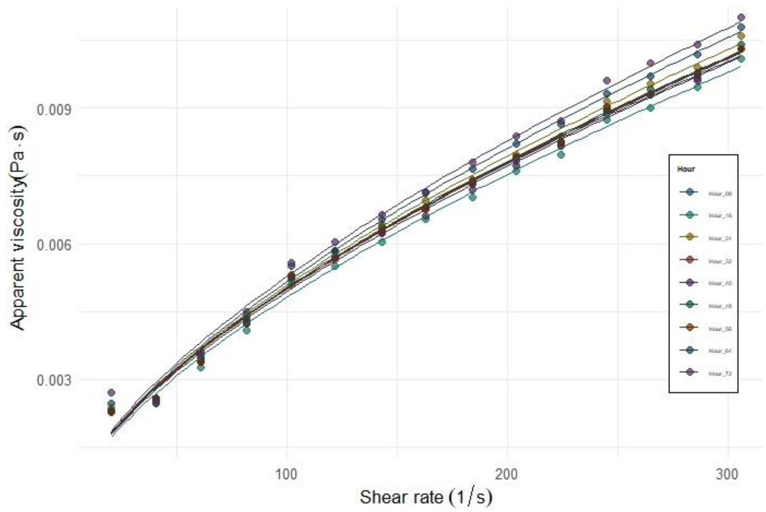

Figure S20. Apparent viscosity curves fitted to the power law model for treatment 1, evolution during fermentation.

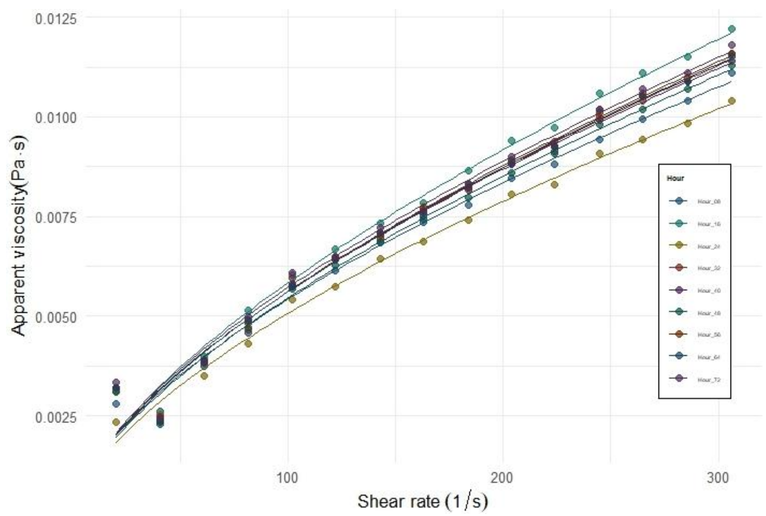

Figure S21. Apparent viscosity curves fitted to the power law model for treatment 2, evolution during fermentation.

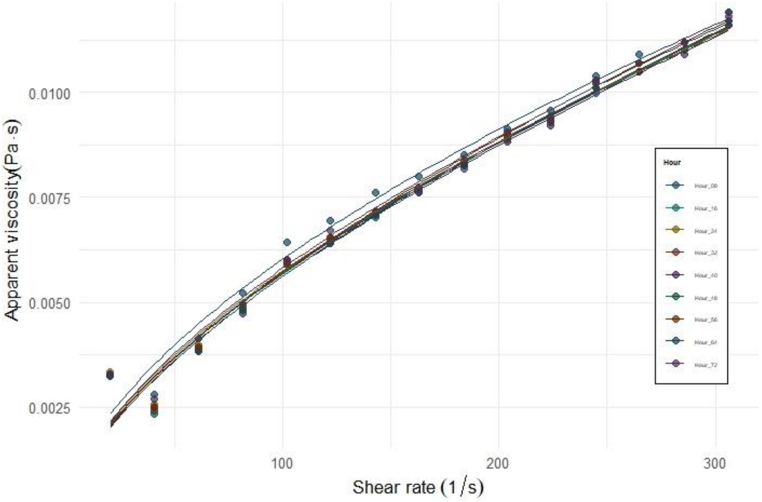

Figure S22. Apparent viscosity curves fitted to the power law model for treatment 3, evolution during fermentation.

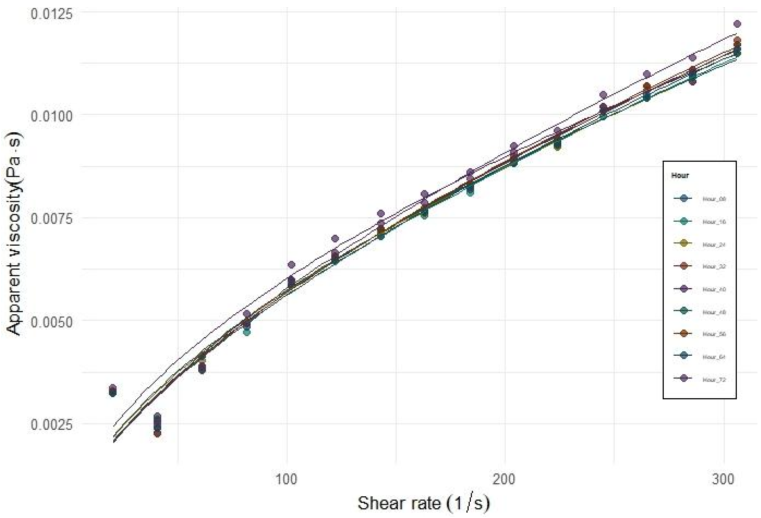

Figure S23. Apparent viscosity curves fitted to the power law model for treatment 4, evolution during fermentation.

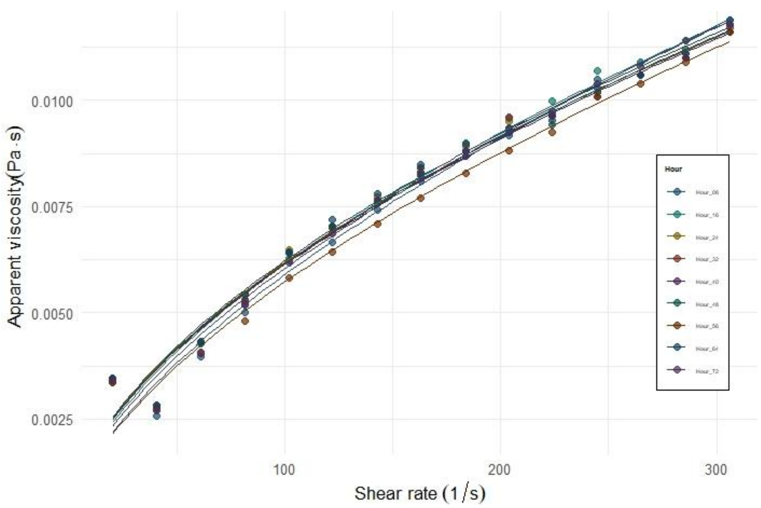

Figure S24. Apparent viscosity curves fitted to the power law model for treatment 5, evolution during fermentation.

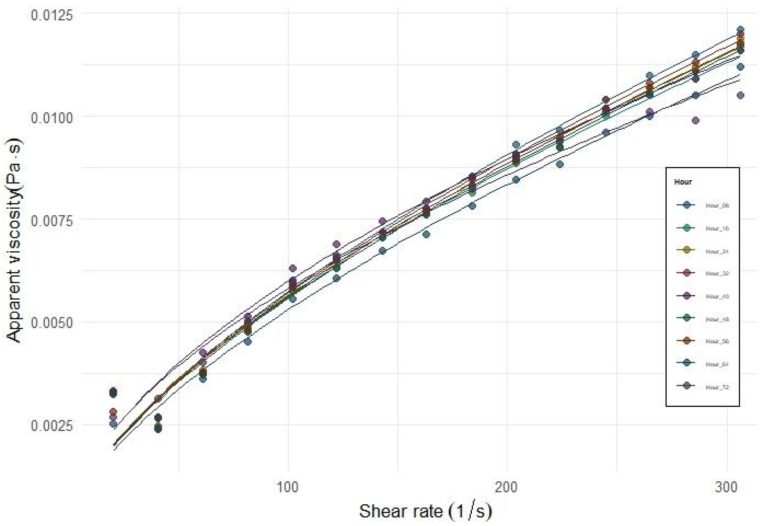

Figure S25. Apparent viscosity curves fitted to the power law model for treatment 7, evolution during fermentation.

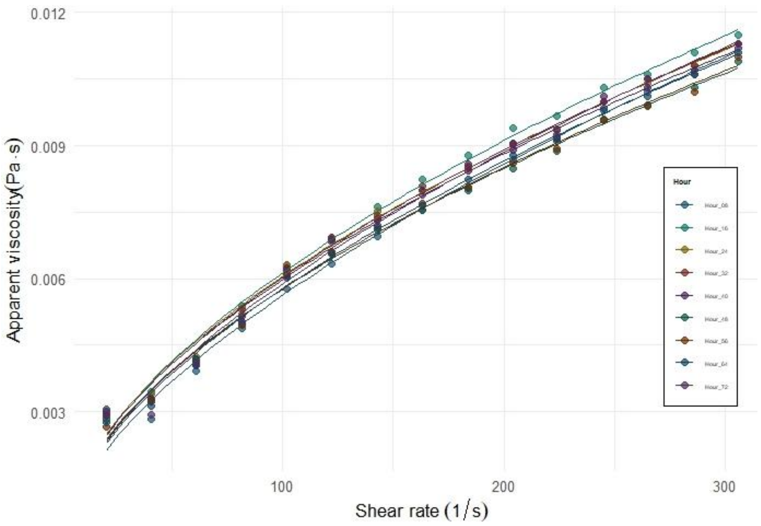

Figure S26. Apparent viscosity curves fitted to the power law model for treatment 8, evolution during fermentation.

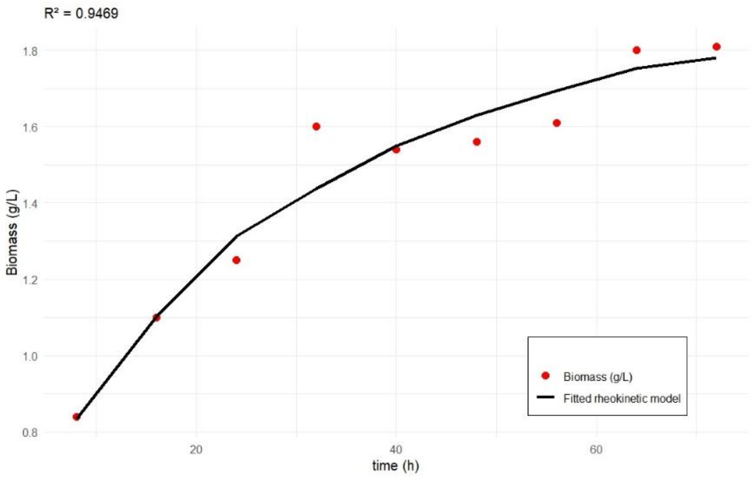

Figure S27. Growth kinetics of biomass modeled from rheological parameters of treatment 1.

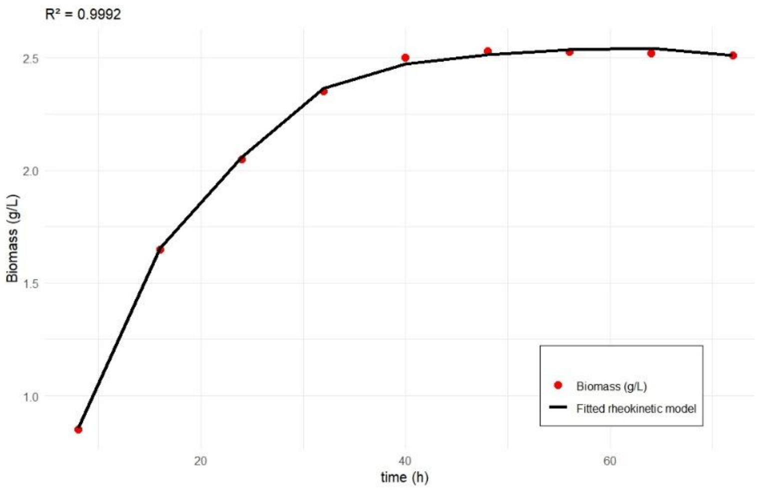

Figure S28. Growth kinetics of biomass modeled from rheological parameters of treatment 2.

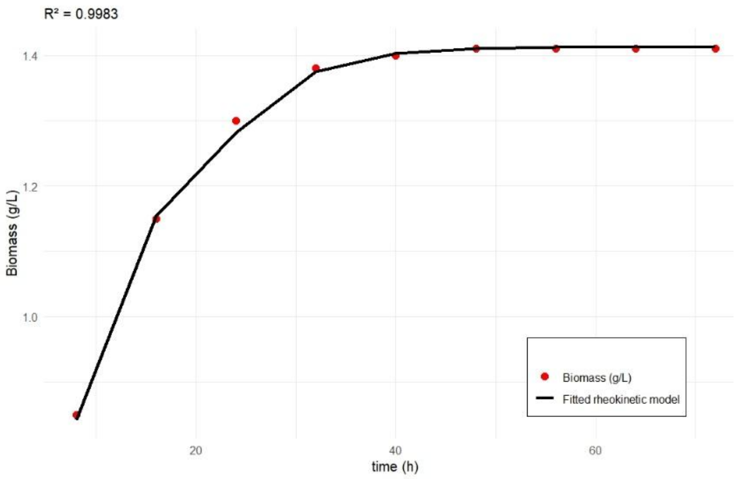

Figure S29. Growth kinetics of biomass modeled from rheological parameters of treatment 3.

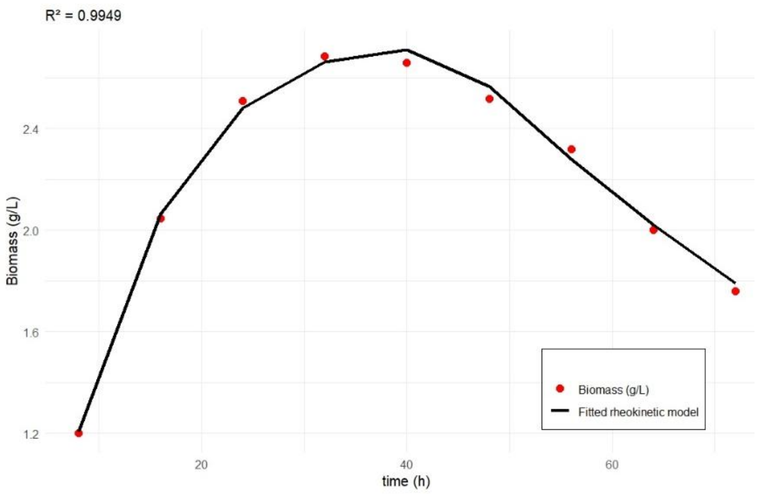

Figure S30. Growth kinetics of biomass modeled from rheological parameters of treatment 4.

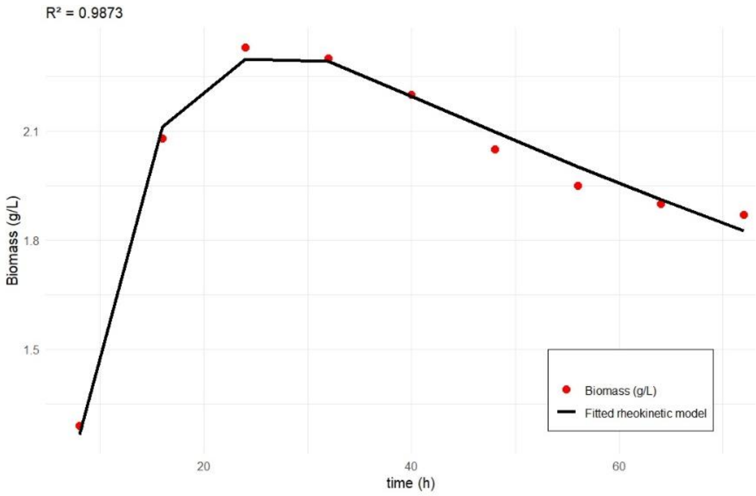

Figure S31. Growth kinetics of biomass modeled from rheological parameters of treatment 5.

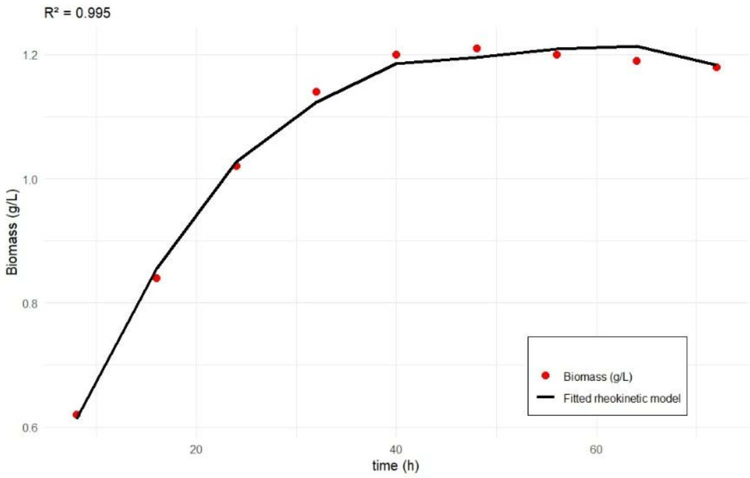

Figure S32. Growth kinetics of biomass modeled from rheological parameters of treatment 7.

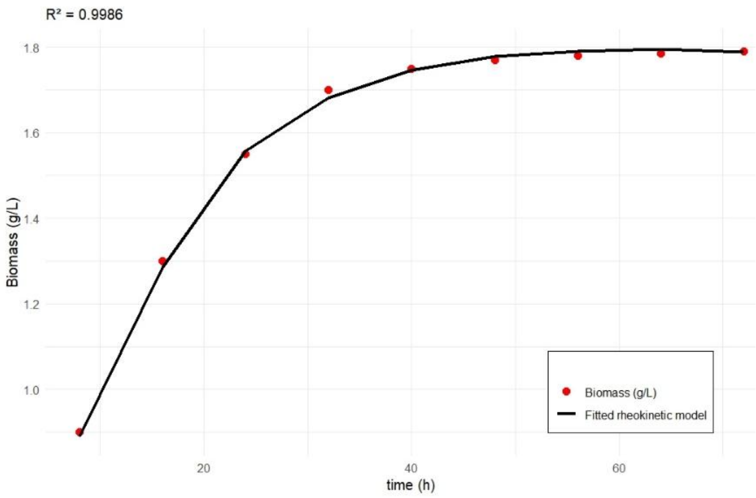

Figure S33. Growth kinetics of biomass modeled from rheological parameters of treatment 8.

Table S1. Correlation coefficients of the Gompertz, Luedeking-Piret, and Pirt models

Treatment	R^2		
	Modified Gompertz model	Luedeking-Piret model	Pirt model
1	0.8956	0.9206	0.9961
2	0.9986	0.9978	0.9986
3	0.8119	0.9749	0.9491
4	0.6603	0.9983	0.9679
5	0.8340	0.9980	0.9936
6	0.9337	0.9917	0.9928
7	0.7855	0.9952	0.9956
8	0.9189	0.9966	0.9976

R^2: coefficient of determination.

Table S2. Rheological parameters of the power law model - treatment 1

Time / d	K / (Pa s^n)	n	R^2
8	2.72×10^{-4}	1.6325	0.9947
16	2.46×10^{-4}	1.6461	0.9940
24	2.63×10^{-4}	1.6431	0.9945
32	2.62×10^{-4}	1.6403	0.9950
40	2.57×10^{-4}	1.6419	0.9938
48	2.68×10^{-4}	1.6369	0.9957
56	2.63×10^{-4}	1.6396	0.9947
64	2.57×10^{-4}	1.6516	0.9928
72	2.64×10^{-4}	1.6504	0.9892

R^2: coefficient of determination; K: consistency coefficient; n: rheological behavior index.

Table S3. Rheological parameters of the power law model - treatment 2

time / d	K / (Pa s^n)	n	R^2
8	3.08×10^{-4}	1.6229	0.9895
16	2.91×10^{-4}	1.6515	0.9854
24	2.69×10^{-4}	1.6375	0.9942
32	2.98×10^{-4}	1.6373	0.9837
40	3.06×10^{-4}	1.6313	0.9815
48	2.82×10^{-4}	1.6430	0.9823
56	2.90×10^{-4}	1.6435	0.9825
64	2.95×10^{-4}	1.6395	0.9814
72	3.05×10^{-4}	1.6360	0.9794

R^2: coefficient of determination; K: consistency coefficient; n: rheological behavior index.

Table S4. Rheological parameters of the power law model - treatment 3

time / d	K / (Pa s^n)	n	R^2
8	3.11×10^{-4}	1.6307	0.9812
16	3.14×10^{-4}	1.6297	0.9814
24	3.30×10^{-4}	1.6204	0.9820
32	3.13×10^{-4}	1.6300	0.9825
40	2.97×10^{-4}	1.6381	0.9810
48	2.88×10^{-4}	1.6455	0.9804
56	2.99×10^{-4}	1.6407	0.9814
64	3.84×10^{-4}	1.5979	0.9843
72	3.32×10^{-4}	1.6216	0.9843

R^2: coefficient of determination; K: consistency coefficient; n: rheological behavior index.

Table S5. Rheological parameters of the power law model - treatment 4

time / d	K / (Pa s^n)	n	R^2
8	3.01×10^{-4}	1.6376	0.9795
16	3.05×10^{-4}	1.6327	0.9815
24	3.39×10^{-4}	1.6130	0.9827
32	2.93×10^{-4}	1.6432	0.9789
40	4.24×10^{-4}	1.5764	0.9806
48	3.48×10^{-4}	1.6085	0.9834
56	3.04×10^{-4}	1.6361	0.9803
64	2.95×10^{-4}	1.6399	0.9809
72	2.86×10^{-4}	1.6523	0.9804

R^2: coefficient of determination; K: consistency coefficient; n: rheological behavior index.

Table S6. Rheological parameters of the power law model - treatment 5

time / d	K / (Pa s^n)	n	R^2
8	3.28×10^{-4}	1.6277	0.9813
16	4.09×10^{-4}	1.5884	0.9806
24	4.60×10^{-4}	1.5641	0.9789
32	4.52×10^{-4}	1.5673	0.9787
40	4.47×10^{-4}	1.5686	0.9816
48	4.44×10^{-4}	1.5707	0.9809
56	3.38×10^{-4}	1.6145	0.9830
64	4.61×10^{-4}	1.5652	0.9796
72	3.78×10^{-4}	1.6020	0.9817

R^2: coefficient of determination; K: consistency coefficient; n: rheological behavior index.

Table S7. Rheological parameters of the power law model - treatment 6

time / d	K / (Pa s^n)	n	R^2
8	3.59×10^{-4}	1.6117	0.9832
16	3.46×10^{-4}	1.6129	0.9840
24	2.75×10^{-4}	1.6507	0.9809
32	2.87×10^{-4}	1.6464	0.9830
40	3.53×10^{-4}	1.6116	0.9839
48	4.22×10^{-4}	1.5803	0.9808
56	4.01×10^{-4}	1.5939	0.9833
64	4.18×10^{-4}	1.5784	0.9822
72	4.36×10^{-4}	1.5763	0.9795

R^2: coefficient of determination; K: consistency coefficient; n: rheological behavior index.

Table S8. Rheological parameters of the power law model - treatment 7

time / d	K / (Pa s^n)	n	R^2
8	2.71×10^{-4}	1.6625	0.9813
16	2.93×10^{-4}	1.6403	0.9801
24	2.82×10^{-4}	1.6505	0.9807
32	2.79×10^{-4}	1.6548	0.9809
40	4.07×10^{-4}	1.5835	0.9828
48	2.71×10^{-4}	1.6560	0.9810
56	2.77×10^{-4}	1.6540	0.9913
64	2.58×10^{-4}	1.6555	0.9936
72	4.35×10^{-4}	1.5628	0.9852

R^2: coefficient of determination; K: consistency coefficient; n: rheological behavior index.

Table S9. Rheological parameters of the power law model - treatment 8

time / d	K / (Pa s^n)	n	R^2
8	3.38×10^{-4}	1.6107	0.9890
16	4.44×10^{-4}	1.5702	0.9937
24	4.63×10^{-4}	1.5579	0.9948
32	4.49×10^{-4}	1.5635	0.9934
40	4.58×10^{-4}	1.5578	0.9922
48	4.45×10^{-4}	1.5564	0.9944
56	4.31×10^{-4}	1.5627	0.9944
64	3.97×10^{-4}	1.5815	0.9953
72	3.95×10^{-4}	1.5865	0.9907

R^2: coefficient of determination; K: consistency coefficient; n: rheological behavior index.